A queuing model to address waiting time inconsistency in solid-organ transplantation

# Abstract

Due to blood compatibility rules, recipients of blood type O experience longer waiting times than those of other blood types, partly due to cross-transplantation of too many O organs to compatible donors of other blood types. Recent developments insist upon ABO-identical transplantation. This study shows that ABO identical transplantation cannot achieve equity. It then presents a model for restricted cross-transplantation which indicates how comparable waiting times for all blood types could be achieved.

# Introduction

Several prior studies assessing waiting times for organ transplantation have revealed that recipients of blood type O wait on average substantially longer than blood type A, while those of blood type A in turn wait longer than patients with blood type AB, regardless of organ type.

Investigators also reported statistically significant increased rates of mortality and graft failure for type-O recipients.

The present work aims to address the impact of ABO-identical and ABO-compatible transplantation upon patient waiting times from the perspective of a new queuing construct, which we name the ‘‘array of idealised transplant queues’’ (AITQ): a model in which donor organs of a particular type can be used for recipients of specified compatible blood groups, and as such, some of the four waitlists or queues are linked.

We establish results which show that the time on the waitlist in such an idealised system is inversely proportional to the rate at which donor organs become available. The significance of this fact is that, in an AITQ operating under an ABO-identical policy, patients from the rarer blood groups of a given region would wait on average longer than the common blood types.

We present what is required, from the perspective of the AITQ model, to achieve comparable waiting times for all blood groups. Limited amounts of cross-transplantation must be allowed to achieve equity, but only between specified blood types, based uniquely upon the blood mix of the given jurisdiction.

# Waiting times for an array of idealised transplant queues operating under an ABO-identical protocol

## Background on donor organ and recipient placement distributions

In transplant queues, the service time constitutes the time between consecutive deceased donor organs becoming available. The process of deceased donor organs becoming available was adequately approximated by a Poisson process. The Poisson-arrivals assumption is widely borne out in large populations (say, of size *n*), in which individual members of the population each have a rare chance *p* of manifesting a particular characteristic. Since inter-event times from a Poisson process are exponentially distributed, this means that we are free to consider the time between consecutive deceased donor organs becoming available to be exponentially distributed.

In contrast, patient placements on the waiting lists are not Poisson. There are several subsequent steps involved prior to placement on a transplant waiting list: the patient’s decision to seek treatment, the consult with their primary care provider, the referral to the specialist, and possible further delays. The length of the waiting list, at any time the decision to place is being considered, might itself play a role. In some cases, a geometric distribution for the daily number placements was found to be suitable.

This combination of a non-Poisson arrival process and exponential service times would seem to suggest the use of a GI/M/1 queue for modelling purposes.

## The array of idealised transplant queues (AITQ) model

Idealised transplant queue model which reflects the most important factors that affect waiting time.

* Patients are placed on the waitlist for patients of blood-type according to a renewal process.
* denotes the random variable representing the time between successive placements for waitlist *i*;
  + The distribution of time between successive patient placements for the *i*-th waitlist is given by 🡪 .
  + The patient placement process has reached stationarity (there is no growth from year to year in the long-term rate at which patients are added to the list).
* Patients on the -th waitlist are served in FCFS fashion, and there is a single server, representing the organ availability process of the -th blood type.
* The times between successive organs of the same blood type becoming available are exponentially distributed at rate .
* We assume that each blood group has the same propensity to donate cadaveric organs and the same need for transplant and as such, each queue can be viewed as a time-scaled version of the others.
* We assume that the (common) long-run cadaveric supply for the four blood types is enough to meet the demand: 🡪
* The moment generating functions for the inter-placement time distributions defined by 🡪
* It is possible to re-assign service capacity (O 🡪 A, B, and AB; A 🡪 AB; B 🡪 AB). This happens with probability each time an organ of type *i* becomes available, where *j* indicates the blood type of the compatible recipient class. We presume that the amount of redirected organs is sufficiently small that each of the queues remains stable (the permissible will be on the order of for any donor class in total).

AITQ as formulated differs from transplant queues in reality for several aspects:

* the choice of an FCFS service discipline for the idealised model, when in many real situations sicker patients gain access to transplantation quicker than healthy ones.
* long waiting times lead to a greater incidence of degraded health and death. A model incorporating abandonments would violate the queue structure we use. Our focus here is to seek a cross-transplantation mechanism that can provide comparable access to patients of all blood types in a timely manner.
  + as the need for transplants among the population at large is likely to resemble a Poisson process, an array of idealised models would reflect the abandonment processes more accurately.
* when prioritised systems are in effect in a system without abandonments, the overall average waiting time for a given blood type is unaffected, as guaranteed by the conservation law. In other words, while a prioritised system will provide better service to the patients most in need, there is no net benefit to a larger fraction of patients as a whole.

The dominant research goals here are:

* establish that an ABO-identical policy cannot be maintained in the long run on the grounds of equity and fairness, for it will lead to shorter waiting times for larger blood groups than for the others;
* establish that a model with restricted amounts of cross-transplantation can do so, producing the desired balance yielding comparable waiting times for all classes.

The process of placing patients on the waitlist is certainly influenced by the human interaction, so the assumption of successive placement times being independent of each other may in fact be false, but it is necessary for reasons of tractability and transparency of the results.

The assumption that the long-run cadaveric supply for the four blood types is sufficient to meet the demand is difficult to assess in a simple manner. Certainly, at present notable numbers of patients die while waiting for many organ types in many jurisdictions, and it is likely the case that if none did, the waitlists would grow over time. This assumption is needed in order to invoke the stationary results for queues that we do.

## Waiting times on the AITQ waitlists

Each of the queues comprising the AITQ features a renewal process for patient placements and has an exponential service mechanism for the organ inter-availability times. Each waiting list can be viewed as a stable queue, so it follows that the sojourn time a patient spends on the relevant wait list is exponentially distributed.

Defining to be the time from arrival until completion of service for a randomly selected customer in the queue, the sojourn time distribution satisfies:

🡪 ,

where is the unique solution to the implicit equation .

Hence the average sojourn time (patient’s average time on the waitlist) is given by:

🡪 .

A notable consequence of these is that any mechanism that manages to equate average waiting times between two components of the AITQ (for instance, type O and type B) will also succeed in equating the percentiles of the waiting time distributions for all .

## Assessing waiting times under ABO-identical transplantation

Under an ABO identical policy, each of the four blood groups can be considered as its own transplant queue. We have assumed that:

* there is no variation in the propensity to donate from one blood group to another,
* the rate of patients requiring transplantation is the same from blood group to blood group.

These imply that the donor rates and arrival rates are in the same proportion for all blood groups. The inter-placement time distributions are therefore scaled versions of each other with the same scale factor that links the donor rates, which is a condition (3) of the theorem that follows.

The same conclusion for the case of blood groups O and B results for type-A and type-AB patients if the same condition is satisfied. Let and denote the respective roots for the separate blood type systems.

**Theorem**.

* Let be the ratio of organ availability rates for blood types B and O respectively.
* Assume that:
  + the moment-generating functions for the patient inter-placement time distributions and exist,
  + the patient inter-placement time distributions are scaled by the same factor that links the donor rates ():

🡪 .

🡪 .

**Lemma**

Let be the common value of and under the assumptions of the theorem. Then the average waiting times for type-B patients will be times larger than those of type O;

🡪 .

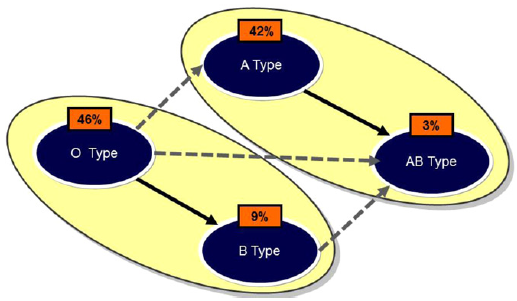
The consequence is that under an ABO identical policy, since O organs become available at roughly

five times the rate that B organs do in the Canadian context, and fifteen times the rate that AB organs, the average waiting time for blood type B and blood type AB patients will be, respectively, about five and fifteen times as long as that of blood O patients. Hence, an ABO-identical strategy is an unsustainable model if equity of access for all blood groups is an aim of the allocation policy.

# Proposed model for limited cross-transplantation

The literature has shown that uncontrolled cross-transplantation adversely affects the blood type O population, but an ABO-identical policy is inequitable on the other hand. An equitable policy must lie between these extremes: it is necessary to allow some cross-transplantation to occur, but one must strive to keep its occurrence less frequent than under existing ABO-compatible policies, in order to provide comparable access to organs for all blood types.

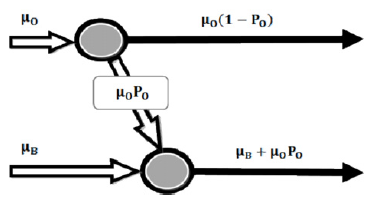
The process of identifying the donor and recipient blood types for permissible cross-transplantation is based upon the following logic that applies in the Canadian context:



1. Group B deceased donor organs do not arise quickly enough to provide the same timely access for a transplant as the larger groups, so they must be supplemented by some amount of cross-transplanted organs. The only compatible source is group 0.
2. Similarly, group AB must be supplemented by some cross-transplanted organs. While all blood groups are compatible to type AB, donor organs of types A and B are preferred to those of type O.
3. Transplants for AB recipients from blood type B would only lead to more transfers from O to B to ensure timely access for type B patients. Hence, the logical source for the type AB recipients is blood group A.

In context where blood mix includes almost equal proportions of O, A, and B type blood, the details of the solution using this approach would need to be modified accordingly.

The resulting policy is a modification of an ABO-identical policy which allows for a small fraction of type O organs to be transplanted into type B recipients, and another small fraction of type A organs to be transplanted into type AB recipients.



* The process of O organs retained for O-type recipients is a Poisson process at rate .
* The process of O organs made available for B-type recipients is a Poisson process at rate (so long as the chance any given organ is selected for cross-transplantation remains at ).
* The resulting aggregate process of deceased donor organs made available to type-B recipients is a Poisson process at rate .

The goal of ensuring fair access is achieved by equating the mean sojourn times and ; this in turn will lead to the same probabilities of waiting time units for a transplant. The resulting equation is:

🡪 .

Solving this equation for in terms of the other parameters leads to the equation:

🡪 , where is the ratio of the rates at which deceased donor organs of type O and type B become available.

In the Canadian context in which , this means that: 🡪 .

Proceeding in the same way for , since , we find that: 🡪 .

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Descrizione generata automaticamente*

The optimal value as a function of the occupancy level for:

* deterministic arrivals,
* Poisson arrivals,
* a particular hyper-exponential inter-arrival time distribution, featuring balanced means and a squared coefficient of variation of 3.77.

Working with an estimate of , and observing that in the case, we find that and will not exceed 4% and 4.6% of organs, respectively, and in fact are likely to be much smaller, ensuring that cross-transplantation is indeed kept rare.

Since the patient placement process is more variable than a Poisson process, one can use as an upper bound on the frequency of cross-transplantation of O organs to B patients. The specific optimal value will ultimately depend upon a host of other factors, such as the identification of a medically and ethically acceptable allocation mechanism, tested via extensive simulation.

# Discussion

From uniquely a waiting time perspective, the manner in which waiting times should be equalised should be dynamic, periodically allowing or disallowing the O → A and A → AB cross-transplantation so as to keep the waiting times on the waitlists in balance. However, such a perspective ignores all medical considerations beyond waiting time.

It is a non-trivial task to infer what the results would look like for any waitlist in which the patient placement rate exceeds the organ availability rate. What one can say is that any such waitlist only achieves ‘‘stability’’ by patients coming off the waitlist. In this context, one cannot ignore the priority aspects, as sicker patients will gain access sooner. In turn, healthy patients will wait longer, to the point that some of them suffer a degraded health status, and so on. While our model cannot determine mathematically what the ideal cross-transplantation frequency rate should be in such a context, one can nonetheless show that some cross-transplantation will be needed, and it is our opinion that it is likely to entail the pairings we have identified for many countries in Europe and the Americas, and possibly in Africa. As the Asian blood type mix is substantially different, a different arrangement would be needed.